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Ituna: A model of the Solway Firth

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1. INTRODUCTION

During a review of the Directorate's modelling requirements, Falconer's two-dimensional tidal model was identified as being one which could be acquired relatively cheaply and adapted to examine the dispersion of radionuclides in estuaries, particularly those of the eastern Irish Sea. This report documents efforts to set up this model for the Solway Firth and to introduce a two-dimensional sediment transport code. Even though the work on the Solway has been suspended, because there are insufficient data to validate the model, the lessons learnt are worth recording for future use, and could be adapted for example, to the Ribble Estuary.

The main body of the report is divided into four parts, namely those on the equations being solved, the discretisation of the equations in the computer program and the method of solution, and that giving some samples of the unvalidated results for the Solway (Sections 2 - 5).

2. EQUATIONS

DIVAST (Depth Integrated Velocities And Solute Transport) was developed by Roger Falconer for a number of applications. The model purchased by MAFF had been used previously to examine the dispersion of nitrates from sewage works discharging into Poole Harbour, Dorset (Falconer, 1986). It began its development as work for his PhD thesis (Falconer, 1976) and has since had a number of applications, such as with the Humber as well as with laboratory-scale model harbours (Owens, 1986).

The equations which the model solves are the two-dimensional, depth-integrated momentum equations, mass conservation equations and a dissolved-tracer dispersion equation. These are derived from the three-dimensional Navier-Stokes equations by averaging over the vertical dimension and applying suitable boundary conditions at the surface and the sea bed. At the surface, wind stress is assumed to take the form of the square of the wind speed multiplied by a surface drag factor. At the sea bed, stress is assumed to be proportional to the square of the fluid velocity multiplied by a drag coefficient proportional to the Chezy coefficient, which is in turn related to the Manning bed roughness coefficient. Falconer (1986) gives details of these equations including the form of the anisotropic dispersion coefficients.

3. DESCRIPTION OF COMPUTER IMPLEMENTATION

The equations are discretised onto a regular, rectangular, staggered grid. That is, not all variables are defined at the same points on the grid, but instead are defined where they are needed. Obviously, values of variables may be needed at points where they are not defined, so in this case the values are determined by linear interpolation from the surrounding points. The resulting finite difference equations are solved using the Alternating Direction Implicit (ADI) method (see Section 4).

3.1 The grid

The variables are defined on a staggered grid (the Arakawa C-grid) as in Figure 1. The water elevation (η) the solute concentration (S) and Chezy value (bed drag coefficient) (C) are all defined in the centre of a computational cell. The depth (or height) of the sea bed is defined at the corners of the cell, and this needs to be averaged to give the value at the centre. As the grid is set up to conserve mass, the flow across the walls of each computational cell is best described by a flux vector on the centre of each side. Thus, the equations are written in terms of areal fluxes (p and q — each calculated by multiplying depth by the x and y components of velocity, respectively), and not velocities at the mid-points of each side. The velocities can be calculated by dividing by the mean depth at the centre of the side.

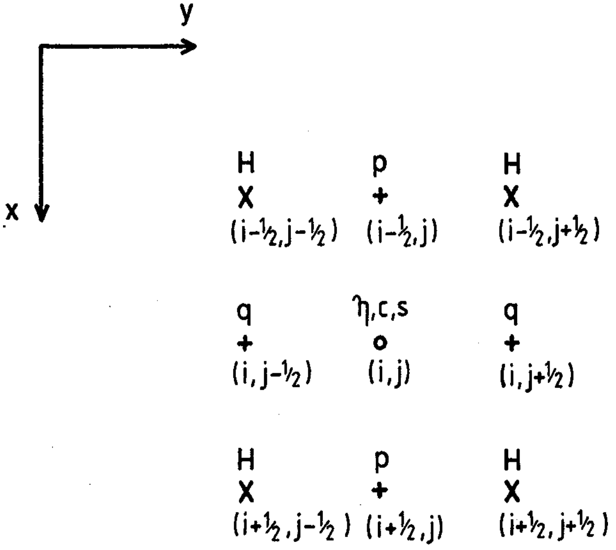


Figure 1. ***The staggered grid for a one-computational cell. When a value of a variable is needed at a location where it is not defined, it is calculated by linear interpolation from adjacent values: where H is the depth of the sea bed below the model datum; p is the water flux in the x-direction; q is the water flux in the y-direction; η is the elevation of the sea surface above the model datum; C is the Chezy coefficient; and S is the solute concentration***

One of the most common mistakes in setting up the model is to have a mismatch between the data being supplied and those which the model is expecting. When writing down the finite differences approximation to the differential equations, it is usual to refer to all discrete values by their position on the grid, as in Figure 1. It should be noted that the indices are usually in half values, whereas the FORTRAN arrays in which these values are stored have integer indices. Figure 2 shows the same Grid as Figure 1, but with the equivalent FORTRAN arrays. Unless this mapping from one grid to the other is carried out carefully, mismatch errors can be introduced.

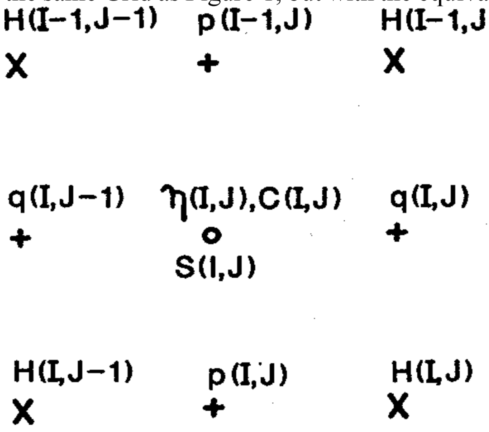


Figure 2. The relationship between the model grid and the FORTRAN arrays (for key, see Figure 1)

DIVAST is written so that the FORTRAN arrays for H, η, C, S, p and q are the same size. Thus, care must be taken near the edge of the grid. For example, DIVAST uses a wet/dry mask to indicate whether computational cells are wet (value 1 and included in the computation) or dry (value 0 and excluded). If there are n cells across each row and m cells down each column, the wet/dry mask array would need nm values. However, each cell requires 4 depth values (one at each corner) so the depth array (H) needs n+1 values across each row and m+1 values down each column. To make the wet/dry mask array the same size as the depth array, a dummy row is added to the 'top' of the array and a dummy column is to the 'left' of the array, as in Figure 3. These dummy values play no part in the computation.

3.2 Specification of the open boundaries

There are two types of open boundary allowed in the model, namely those where an inflow is specified and those where a surface elevation is given. Each of these are sub-divided into lower regions (to the top or left of the model area) and upper regions (to the bottom or right of the model area). The type of boundary must be specified in the input data file together with details of cells which are included. Figure 4(a) shows an upper inflow boundary, with indices referred to the wet/dry mask data and Figure 4(b) shows a lower elevation boundary. On an inflow boundary, the total flow across

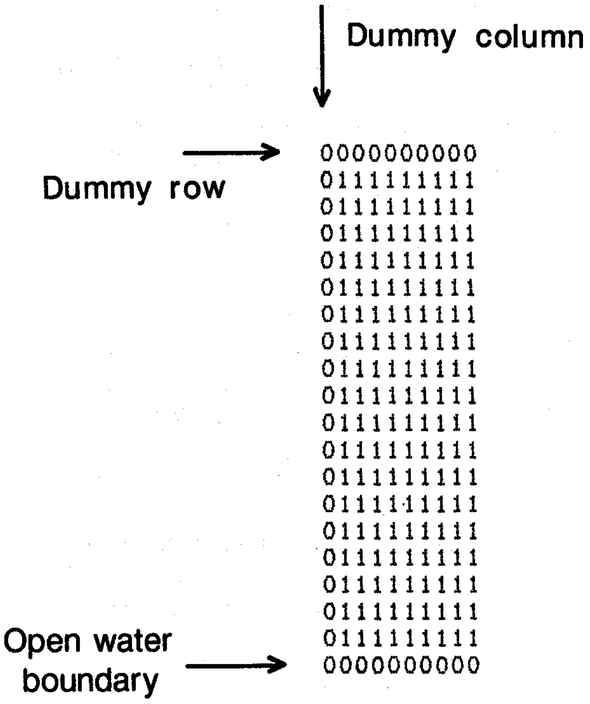
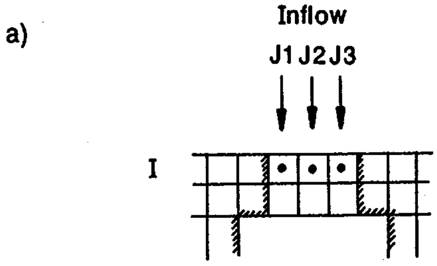
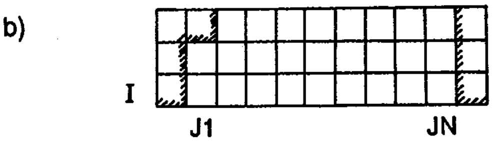


Figure 3. A typical wet/dry mask for the model computations. The lines of zeros along the left-hand edge and top are needed to allow FORTRAN arrays to be in the same dimension. Zeros along the bottom or right-hand edge are there to indicate land (a closed boundary) or water (an open boundary). The only way of telling which variable they represent is to examine the topographic data



Inflow boundary Row I Columns J1-J3



Elevation boundary Row I Columns J1 -JN

Elevation is specified at J1 and calculated for J2-JN taking into account the Coriolis effect

Figure 4. Specification of the extent of the boundary: (a) an upper inflow boundary; and (b) a lower elevation boundary

the whole boundary needs to be specified. If this region occupies more than one grid square, then the flow is spread uniformly across the boundary. On an elevation boundary, the tidal height is specified. If no detailed information is available across the whole boundary, then the elevation should be specified at one point and then 'spread' across the whole boundary taking account of the Coriolis force.

Open boundaries where tidal heights are defined, such as at the mouth of the estuary, should be indicated as dry cells when defining the wet/dry mask but depths also need to be defined for these cells. The code takes note that these are special cells and acts accordingly.

3.3 Specification of sea bed topography

All water depths, and drying area heights need to be referred to a standard level. Normally, when setting up the model for a specific area, the water depths are obtained from a suitable chart (usually an Admiralty Chart). The easiest reference level to use is the chart datum level. On Admiralty Charts, this is usually the level of the lowest astronomical tide. However, care must be taken in referring the tidal height data on the open boundary to the same reference level.

Normally, when a model is set up, the relative cost of computer time has to be balanced against the spatial resolution of the model. Ideally, the model grid should resolve all of the main features. However, no matter which scale is chosen, there are frequently small-scale creeks and gullies of which account needs to be taken. For example, in the upper reaches of an estuary which dries out at low tide, narrow creeks take the river flow to the deeper water. Such creeks are almost inevitably narrower than the grid spacing. If they are omitted, there is nowhere for the river to flow and, if they are 'widened' to at least one grid cell width, the volumes which they allow are larger than reality. Some compromise has to be reached which is usually a wider, shallower representation.

Flather and Hubbert (1990) have examined a number of ways of dealing with narrow creeks in a model of Morecambe Bay. They conclude that if the effect of the creek is to be included then it should be artificially widened to a one-computational cell and the depth adjusted to allow for the correct net flow.

It is worth noting that, when the model grid has been laid upon a suitable chart, there are often areas where the water depths are ill-defined. These are areas which dry out at low tide, or may even change from time to time (such as a deep channel between sand banks). Under these circumstances, estimates of depth should be obtained from other sources (preferably from a bathymetric survey but could be from satellite or aerial photographs).

When estimating the depth at a grid point, the way in which the value will be used should be borne in mind. Most frequently, the value will be averaged to give the average depth of the cell. Alternatively, it will be averaged to give the mean depth of the side of a cell. The latter is especially important in deciding which cells are dry and which are flooded.

This version of the model has the water depth defined at the corners of each cell (see Figure 1). An alternative method would be to define depths at the mid-points of each side of the cell where the water fluxes (p and q) are defined. The model could be rewritten with this information but more data would be needed. For a grid of n x m cells, (n + 1) x (m + 1) depth values are needed if the depths are defined at the corners and n x (m + 1) + (n + 1) x m values are needed at the mid-points of cell sides - a difference of mn + 1 values.

3.4 Orientation of the grid

Many models assume that the model grid runs north/south and east/west. This model can be set up with any orientation, which is particularly useful when modelling an irregularly shaped estuary. The model grid can be orientated so that there is a maximum of 'wet' cells compared with 'dry' cells (or land). All that needs to be specified is the angle measured anti-clockwise from the x-direction of the model to the north.

3.5 Flooding and drying algorithm

In many estuarine models, account must be taken of the flooding and drying of intertidal areas. An inappropriate method of determining when a model grid cell is wet or dry can cause numerical difficulties and introduce errors into the solution. The procedure adopted here was based on that of Leendertse (Leendertse, 1970; Leendertse and Gritton, 1971) but modified in two ways, to improve computational efficiency and to increase the accuracy of the scheme (Falconer, 1986). Flather and Hubbert (1990) have recently reviewed some flooding and drying techniques for tide and surge models.

4. METHOD OF SOLUTION - ADI

The method of solution of the finite difference equations is always to some extent a matter of personal preference. Each numerical scheme has a different set of advantages and disadvantages, some of which may change with computer architecture. For example, the Alternative Direction Implicit (ADI) method works well on one processor, but there can be serious problems with data communications between computational cells when trying to implement ADI on an array of transputers. The essence of the method is that each time step is split into two. During the first half of the time step, an implicit scheme is used to calculate the flows in the x-direction. That is, the finite difference equations are written so that all of the terms which involve p are at time n + ½, whereas all terms involving q are at time n. This leads to a set of linear equations which can be solved for the unknown p values. As the matrix of the coefficients of these equations is tri-diagonal, Gaussian elimination and back substitution is used. At the next half time step, the same procedure is repeated for q in terms of p. Figure 5 shows the combinations of points used at each half time step.

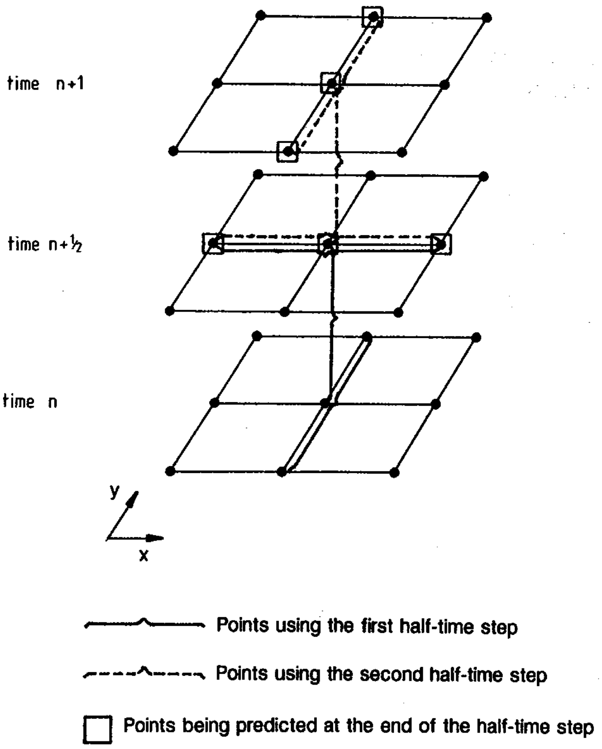


Figure 5. A schematic diagram indicating the combination of variables taking part in the two half time steps in the ADI method

5. RESULTS FROM THE SOLWAY FIRTH

5.1 The topography

The Solway Firth is a large embayment of the north-eastern Irish Sea, fed by small rivers from the surrounding high ground of the Galloway and Cumbrian hills. There are extensive drying areas at low tide, as shown by Figure 6 which is taken from a satellite photograph provided by the National Remote Sensing Centre Ltd, Farnborough (NRSC). Admiralty Charts of the area show that many of these drying areas have unknown heights as many of these sand banks change position. By using information from Admiralty Chart 1346 (Great Britain - Hydrographer of the Navy, 1989) for the region, satellite photographs such as that in Figure 6, the Ordnance Survey map of the upper estuary and other information (e.g. Water Resources Board, 1966), it has been possible to draw together the bottom topography for the model (Figure 7). The resolution used (460 m x 460 m) was selected to show some of the creeks and features of the Solway Firth. In the upper estuary, there has been artificial adjustment of the depth to take account of two of the larger rivers, the Esk and Eden, which flow into the Solway.

5.2 The river flows

There are 8 main rivers entering the Solway: the Urr, Nith, Lochar, Annan, Esk, Eden, Wampool and Waver. None of these has very large flows, as can be seen in Table 1.

Table 1. Average river flows ($m^3 s^{-1}$) provided by the Solway River Purification Board and North West Water

River	Flow	River	Flow
Urr	5	Esk	34
Nith	40	Eden	45
Lochar	5	Wampool	2
Annan	28	Waver	3

5.3 The open boundary and tidal information

The model was forced with tidal heights defined on the open boundary. Many runs were made with simple M_2 forcing using the tidal height data taken from Admiralty Chart 1346 for Workington. There are few available sets of tide gauge data from around the area which makes it difficult to force the model on the open boundary and to validate it. There is a tidal gauge at Workington for which data have been obtained. Figure 8 shows the tidal elevation for the period from 1 March to 1 April 1977. Also, a number of tidal constituents have been determined for a number of locations around the estuary. An indication of existing data is given in Table 2 which was supplied by British Oceanographic Data Centre, Bidston, Merseyside (BODC). Additional data can be obtained from Admiralty Tide Tables (see, for example, that for the year 1990).

Table 2. Availability of tidal constants for Solway Firth

Location	Lat. N	Long. W	Data length	Day (central date)	Month	Year	Pairs of constants
Hestan Islet	54°50'	3°48'	15	11	5	1938	9

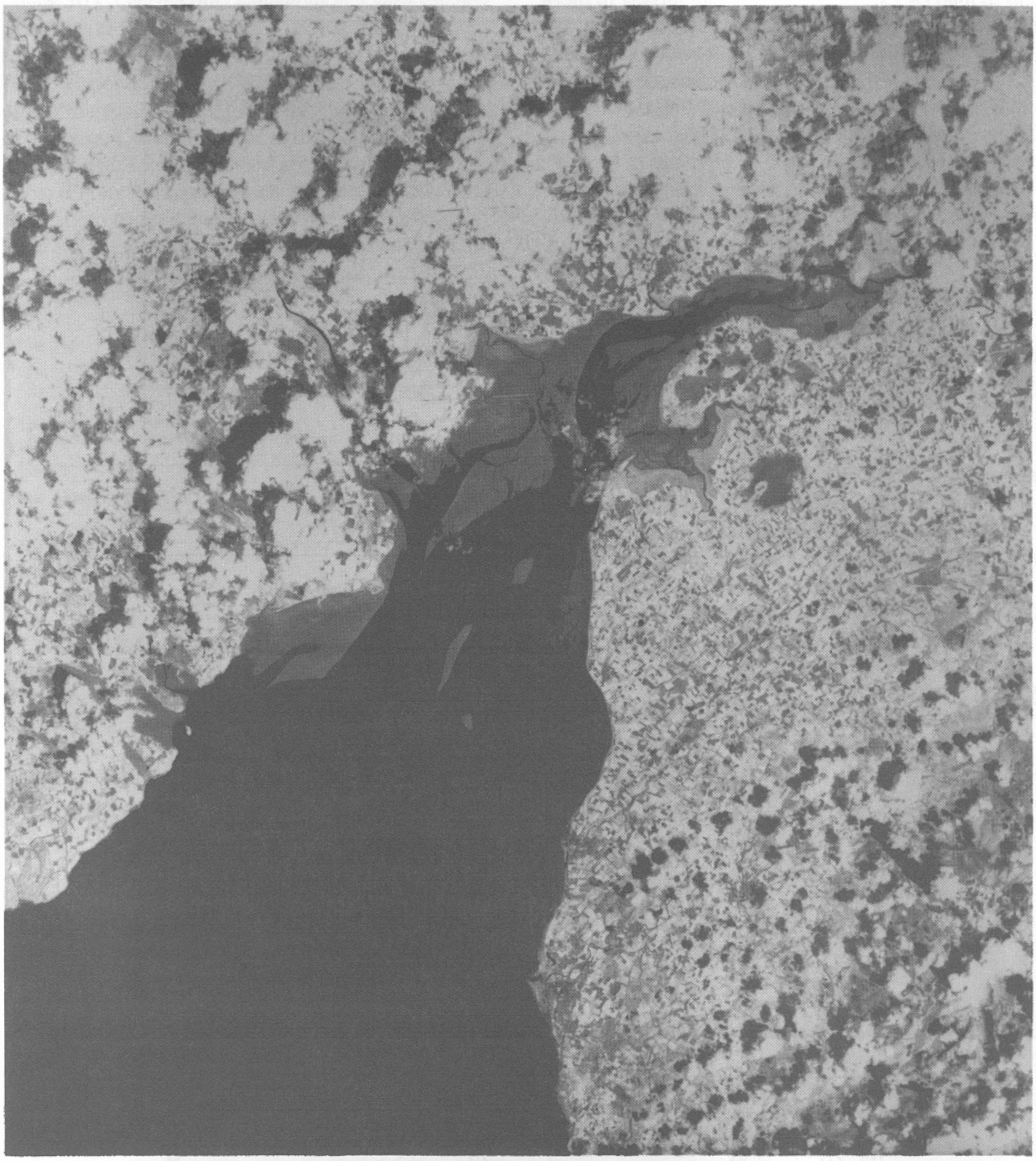


Figure 6. An extract from a satellite photograph (KIFU TM5 205/22 Band 4, © NRSC Farnborough) taken on 16 May 1986. The Solway Firth is almost cloud free but there are a number of clouds over the land, particularly over Galloway. The extensive intertidal area can be seen to be exposed near low tide

Workington	34°39'	3°34'	364	6	5	1975	14

The Solway Barrage Report (Water Resources Board, 1966) provides tidal curves from 9 March 1966 (springs) and 16 March 1966 (neaps) for the same upper estuary locations (Redkirk, Glasson, Newbie and Silloth) and these are reproduced in Figure 9.There are no current meter measurements within the estuary, according to BODC, but there is one MAFF mooring with two instruments at 54°37.9'N 4°28.0'W in 52 m of water just outside the region covered by the model. The bottom meter (8 m up) worked for 10 days and the upper meter (36 m up) for 47 days, during a deployment which began on 26 March 1977.

5.4 Tidal height results

Most of the model runs have been made with the M_2 tidal signal imposed across the mouth of the estuary with the river flows described in Table 1. Normally, the model was started from a state of zero flow and allowed to run for a number of tidal cycles until it reached some sort of periodic state. The results were then examined

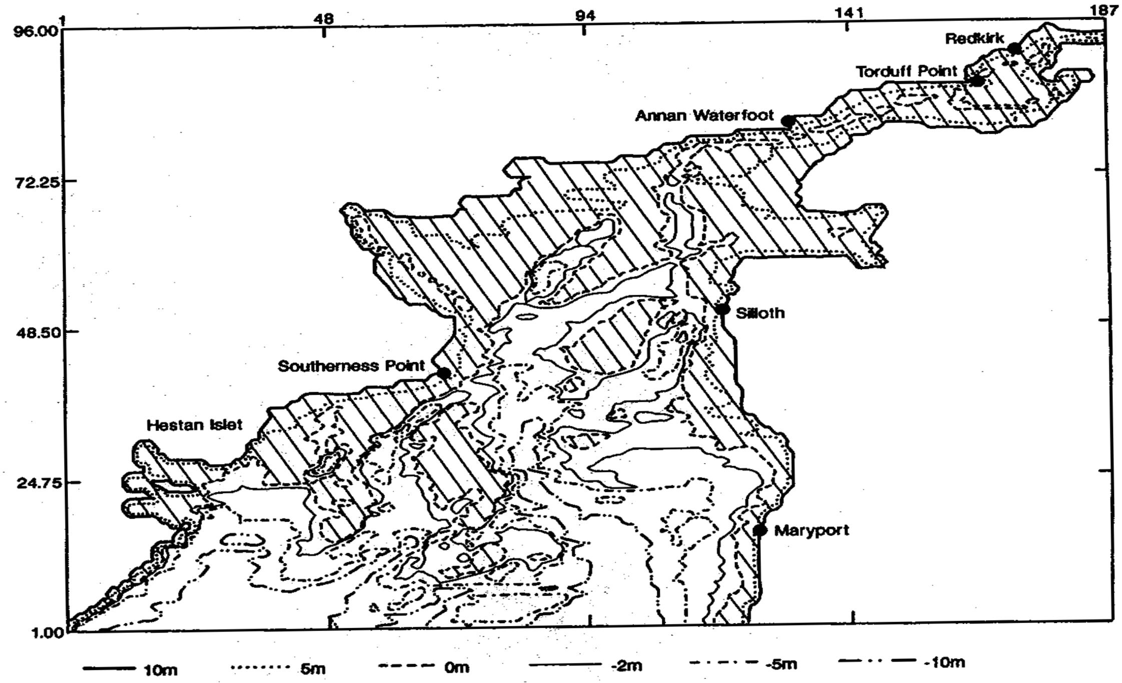


Figure 7. The bottom topography for the model with some locations indicated

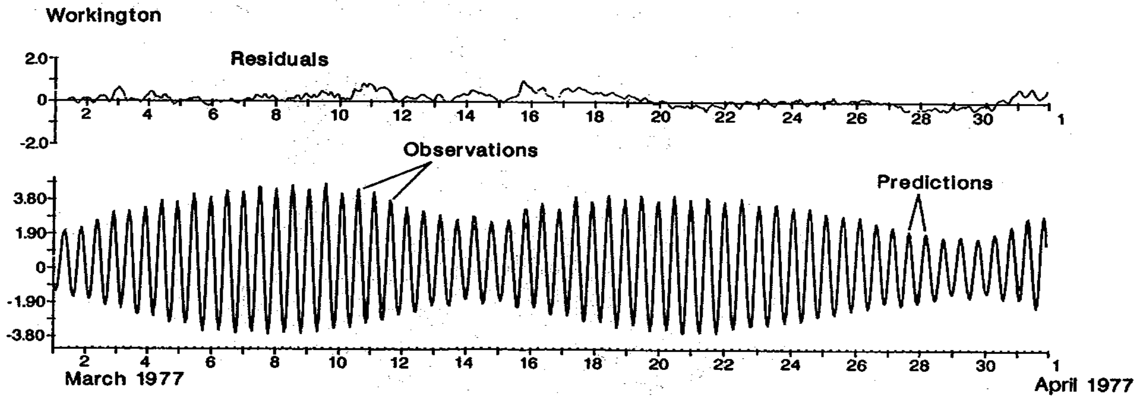


Figure 8. Time series of the tidal elevations at Workington for the period 1 March to 1 April 1977

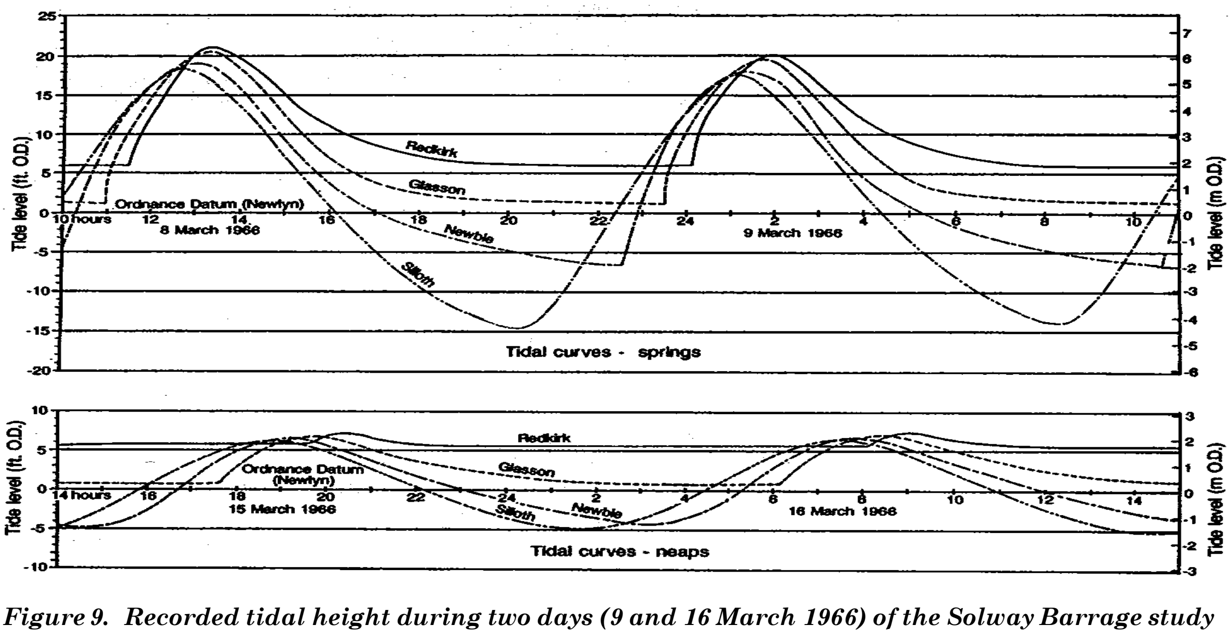


Figure 9. Recorded tidal height during two days (9 and 16 March 1966) of the Solway Barrage study (Water Resources Board, 1966)

for model simulation times between 140 h and 186 h. Figure 10 shows the tidal elevations at 7 sites situated clockwise around the estuary. The curves for model sites at Southerness Point and Maryport have extensive flat sections which indicate that the grid box dries out during the simulation. Hestan Islet is near the mouth of the estuary, and thus close to the open boundary where the M_2 forcing is being applied. The tidal height curve (Figure 10(a)) shows a similar sinusoidal variation. It can be seen that Hestan Islet and Workington are expected to have very similar tidal elevations and the model confirms this observation (also see Table 3).

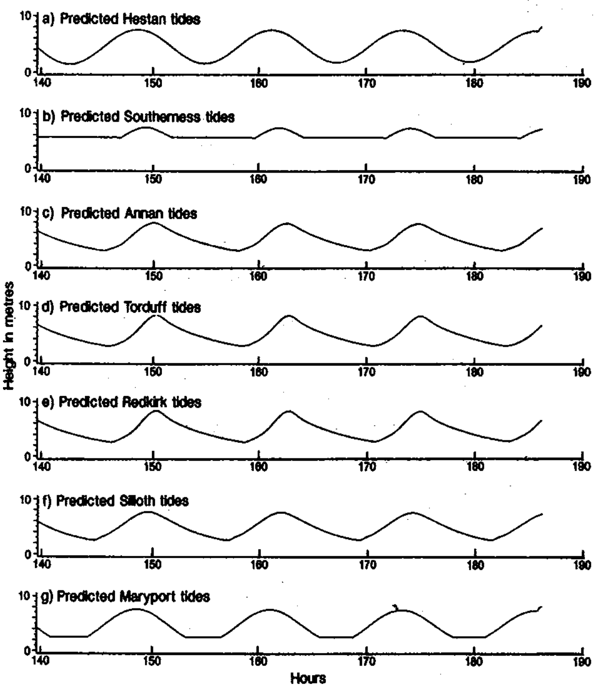


Figure 10. ***Model predictions of tidal elevations at a number of locations (see Figure 7) around the Solway Firth***

Table 3. ***Tidal levels taken from Admiralty Chart 1346***

Location	Lat. N	Long. W	Height in metres above datum			
			MHWS	MHWN	MLWN	MLWS
Hestan Islet	54°50'	3°48'	8.3	6.3	2.4	0.9
Southerness Point	54°52'	3°36'	8.6	6.7	-	-
Newbie	54°58'	3°17'	7.1	4.8	0.2	0.0
Silloth	54°52'	3°24'	9.2	6.9	2.3	0.8
Maryport	54°43'	3°30'	8.6	6.6	2.5	0.9
Workington	54°39'	3°34'	8.4	6.4	2.5	0.9

Further up the estuary there is an asymmetry in the tidal cycle with a more rapid flood than ebb. Also, the time of the peak and trough elevations lag behind those at Hestan Islet. This pattern can also be seen in tidal data from the Solway Barrage study (see Figure 8). The model predicts a rise and fall of about 5 m compared with 4 m from the observations, but the observations indicate that the Redkirk location dries at low tide, whereas the model does not dry, which may be a result of altering the topography to take account of the Esk and Eden river flows. The model prediction at the Silloth site shows a 5 m rise, which is less than 8 m from the observations (see Figure 8) and less than 8.4 m between MHWS and MLWS. Some of the above discrepancies may be accounted for by the influence of the wind in shallow water (the model was run with zero wind) and also by the lack of other tidal components. Further investigation is needed to clarify these points.

5.5 Flooding and drying

The algorithm for flooding and drying of grid cells is described above. It appears to work well, although some indication of the onset of flooding can be seen in the tidal height predictions in Figure 10(b) (the predictions for Southerness Point) with a slight dip in elevation before the rise of the tide.

To view the extent of flooding and drying in the model, some colour graphics software has been developed by MAFF although, until such time as a suitable colour plotter becomes available, it is not possible to reproduce the results in this report. It is possible to include only the wet/dry mask printout (1 is wet, 0 is dry) at low tide

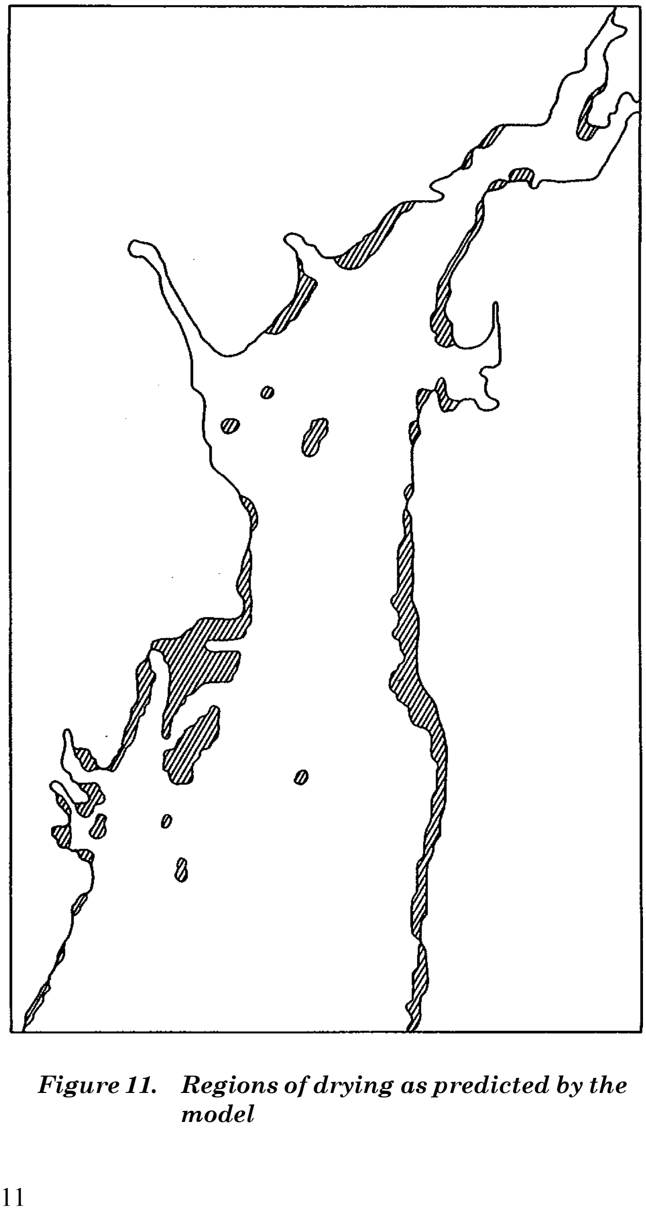


Figure 11. ***Regions of drying as predicted by the model***

at the mouth of the Solway (Figure 11). The shaded regions represent the areas which have dried. Unfortunately, these do not coincide well with the drying areas shown on the satellite photograph (see Figure 6) which indicates that more work is needed on the topography to improve the model. The lower estuary between Hestan Islet and Southerness Point seems to be in reasonable agreement but the drying areas are a different shape in the upper estuary. An attempt has been made to increase the height of the topography in this area automatically, but this has not led to major changes. A closer examination of the topography is required.

5.6 Dissolved tracer dispersion

Falconer (1986) used his model to examine the dispersion of nitrates in Poole Harbour. In this case, all of the inputs were within the Harbour. However, our main interest in modelling the Solway is to determine the likely accumulation in the estuary of radionuclides discharged from Sellafield. This means that the source is at the mouth of the estuary. Originally, it was hoped that the time series of concentrations generated by the MIRMAID model (Gurbutt and Kershaw, 1989) could be used to specify the concentration on the open boundary. In the experiments so far, a fixed concentration is assumed across the boundary, although this can be allowed to vary with time.

The difference scheme for the dissolved tracer is the same as that for the water velocities, namely ADI based on centred spatial differences for gradients. Such a scheme is well known for producing spurious negative concentrations associated with the 'front' of a tracer spreading out from a point source. Simple experiments with such a release have confirmed this point. Changing to an 'upwind' scheme, which has now been implemented, should remove this effect, but it is known that the upwind scheme produces high numerical diffusion. However, the centred scheme appears to be adequate for the case of an imposed concentration across the open boundary, especially if some constant, non-zero value is given to the initial tracer concentration throughout the estuary.

Falconer (1986) found that the tidal and spatially average input of tracer at the boundary (N(t)) could be related to the mean outflow tracer concentration (N) on the previous ebb tide:

$$N(t) = \theta N$$

The constant of proportionality, θ, used for the nitrate simulations in Poole Harbour, was 0.75 and the same constant has been used in simulations of the Solway. However, this relationship is only a half of the boundary condition. In addition, to replacing that which was flushed out on the previous ebb tide, further tracer must be added because the area outside the boundary is assumed to behave like a large reservoir.

Figure 12 shows the flux of water and solute across a line running approximately from Hestan Islet to Maryport. The water fluxes in and out remain constant throughout the experiment, but the tracer fluxes increase. Here, there was a constant source at the mouth of one unit which changed to two units after 140 hours. After an initial adjustment, the solute flux, both inwards (positive) and outwards (negative), increases at a constant rate until the increase in boundary concentration is introduced. The rate of increase in the fluxes then increases to match.

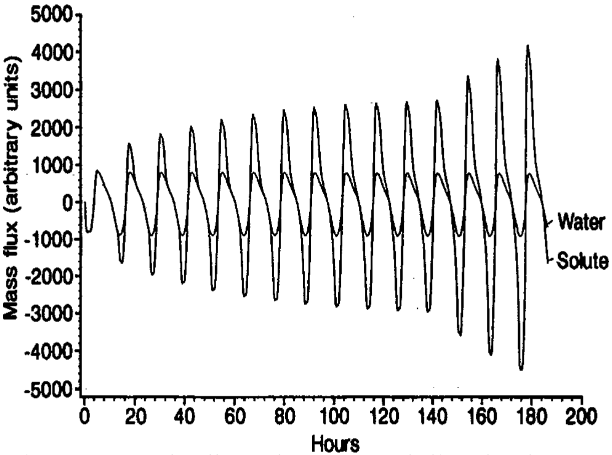


Figure 12. ***The flux of water and dissolved tracer across a line in the model running from about Hestan Islet to Maryport. There is an initial uniform tracer distribution and a steady source across the mouth of the estuary for the first 140 hours of the simulation. The source then doubles over the period of 5 computational steps (450s)***

The colour graphics display will show the tracer being carried in on the flood and being removed on the ebb, which is much as expected.

6. SUMMARY

The Solway Firth model (Ituna) has been a useful learning tool for DIVAST. However, it has been extremely difficult to find data both for tidal heights and currents; and for suspended sediment concentrations and other features. There have been a few measurements of Sellafield-derived radionuclides in the area (Garland *et al.*, 1989; Jones *et al.*, 1984) as well as results from MAFF's monitoring programmes (Hunt, 1989) but one of the main problems in obtaining the data for modelling has been the difficulty of sampling. Jones *et al.* (1984) used a hovercraft fitted with a gamma spectrometer and were able to cover much of the intertidal regions. However, to further extend these radioactivity measurements and to collect physical information such as tide gauge and bathymetric data and suspended loads throughout the tidal cycle, would involve a large field programme which cannot be envisaged at present.

Ideally, a number of tide gauge deployments would be useful at different locations in the estuary together with information on sediment distribution, suspended loads throughout the tidal cycle at a number of locations and data on radionuclide distribution.

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